



USDOT Region V Regional University Transportation Center Final Report

NEXTRANS Project No. 081PY04

Design of Personal Rapid Transit Networks for Transit-Oriented Development Cities

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TECHNICAL SUMMARY

NEXTRANS Project No. 081PY04

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Title

Design of Personal Rapid Transit Networks for Transit-Oriented Development Cities

Introduction

Personal rapid transit (PRT) is an automated transit system in which vehicles are sized to transport a batch of passengers on demand to their destinations, by means of nonstop and non-transfer on its own right-of-way. PRT vehicles run exclusively on its tracks, called guideways. The guideways are designed as elevated facilities above the ground, for purpose of eliminating at-grade crossings or interferences with other transportation modes. In US, PRT has been implemented as a mode of automated people movers at airports and institutions like schools, e.g., PRT system in West Virginia University campus in Morgantown, WV. Worldwide, PRT systems have been designed for several real world applications recently, including in Korea, Sweden, and United Arab Emirates.

In the recent planning practice for urban development in future, there has been an increasing and sustained emphasis in the global community in sustainable transportation systems. Transit-oriented development (TOD) has emerged as a promising alternative for sustainable communities by creating compact environments using convenient and efficient public transportation systems. To facilitate TOD development, an alternative to the personal car needs to provide a public transit mode which offers the same door-to-door flexibility at an acceptable cost. This could be achieved through a mixed design of high passenger-flows mass transit and flexible public transportation carrying low passenger-flows for the times or places. PRT is one of such flexible systems serving a supplement mode for the TOD development, where a PRT system functions as a local area network, connecting the traditional transit systems and other means of transit modes within its network.

Although PRT has been recognized as an important component of alternate solutions to passenger cars in sustainable transportation systems in the future, it has not yet achieved wide-spread commercial deployment in US. Two major downsides that restrict the PRT in the practical stage are the cost and line capacity. Both the cost and line capacity could be improved through an appropriate guideway network (GN) design, because a well-designed GN not only improves the connectivity and accessibility, but also provides more options in the route choice. In this study we investigate the methodology of PRT network design, to minimize both guideway construction cost and users' travel cost. In particular we introduce a set of optional points, known as Steiner points, in the graph to reduce the guideway length. The model is formulated as a combined Steiner problem and assignment problem, and a Lagrangian relaxation based solution algorithm is developed to solve the problem. Numerical studies are carried on a realistic-sized network. We show the proposed model and solution algorithm can solve the PRT guideway network effectively.

Findings

The following findings have resulted from the study. First, we present a model and solution algorithm for the GN design for the PRT supporting transit-oriented development. The GN is designed to balance two objectives, i.e., life-time construction cost and users' travel cost. Second, the Steiner points can reduce the GN length as well as the construction costs substantially. The resultant GN design problem is then formulated as a combined Steiner problem and assignment problem. Third, we propose a Lagrangian relaxation algorithm to decompose the Lagrangian dual problem into two subproblems; both are trivial to solve. Finally, we use a realistic-sized numerical example to demonstrate the computational performance, and validate the fact that a set of Steiner points can reduce both construction cost and users' travel cost significantly.

Recommendations

The research addressed in this project suggests that the proposed model can be used to assist PRT GN design supporting transit-oriented development. The proposed methodology can be applied to offline planning, or what-if scenario analysis. We show that the proposed Lagrangian relaxation algorithm can solve a realistic-size PRT example optimally in an efficient manner.

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CHAPTER 1. INTRODUCTION

1.1 What is Personal Rapid Transit

Personal rapid transit (PRT) is an automated transit system in which vehicles are sized to transport a batch of passengers on demand to their destinations, by means of nonstop and non-transfer on its own right-of-way (Anderson 1998). A PRT system provides a service similar to taxi, because passengers are served on demand, and there are no pre-determined schedules for PRT. At the PRT station, a group of passengers first select the intended destination station; a PRT vehicle is then dispatched to the station to carry the passengers to the desired destination. Stations are offline such that vehicles can accelerate/decelerate on auxiliary lanes without interfering with the vehicles passing by the main through-lanes; hence, a PRT vehicle can run in a non-stop manner via bypassing all intermediate stations (see Figure 1.1). Nowadays, PRT vehicles supported by modern technologies are usually designed running on electricity, and are operated by computer control requiring no driver. The size of a PRT vehicle can accommodate three to six passengers in general (see Figure 1.2).

PRT vehicles run exclusively on its tracks, called guideways (see Figure 1.3). The guideways are designed as elevated facilities above the ground, for purpose of eliminating at-grade crossings or interferences with other transportation modes. In US, PRT has been implemented as a mode of automated people movers at airports and institutions like schools, e.g., PRT system in West Virginia University campus in Morgantown, WV (Sproule and Neumann 1991). Worldwide, PRT systems have been designed for several real-world applications recently, including in Korea (Suh 2001),

Sweden (Tegner, Hunhammar et al. 2007) and United Arab Emirates (Mueller and Sgouridis 2011).



Figure 1.1 PRT Offline Stations



Figure 1.2 PRT Vehicles



Figure 1.3 PRT Vehicles Runs on the Guideway

1.2 Background and Motivation

In the recent planning practice for urban development in future, there has been an increasing and sustained emphasis in the global community in sustainable transportation systems. The excessive use of personal cars has led to several issues, including congestion, energy consumption, air pollution, noise, safety and excessive land use. Transit-oriented development (TOD) has emerged as a promising alternative for sustainable communities overcoming the issues above by creating compact environments using convenient and efficient public transportation systems. TOD is deployed to reduce people's dependence on personal cars for mobility and to help make livable and vibrant communities. The most vital element in a TOD design is the planning and the design of the public transportation network which serves the backbone of urban infrastructure systems (Lin and Shin 2008; Li, Guo et al. 2010). A recent trend in the TOD deployment is to introduce efficient transit systems such as bus rapid transit

(BRT), light rail transit (LRT) and personal rapid transit (PRT) (Parent 2006). Among these modes, PRT has received significant attention as it connects personal, private, and public transportation scales, and because of its flexible operational characteristics and competitive financial investments (Muir, Jeffery et al. 2009). For instance, Tegner et al. (2005) estimated that the construction cost of a PRT system is about a third that of light rail, as PRT has much smaller vehicle size and the lower design standard of guideways than LRT.

Mass transportation systems such as trains, metros and rapid buses represent the major means of TOD development (see Figure 1.4). These modes are efficient in terms of transporting passengers measured by per unit of space or energy, provided that the demand is sufficiently enough. If the demand decreases, however, the ridership drops while the operation cost remains the same. Thus, the system's efficiency degrades. This is a key reason why most mass transit systems reduce frequencies during off-peak hours (Clerget, Hafez et al. 2001). Hence, each mass transit system has a certain operating range in terms of passengers per hour to maintain an efficient operation of the system. To facilitate TOD development, an alternative to the personal car needs to provide a public transit mode which offers the same door-to-door flexibility at an acceptable cost. This could be achieved through a mixed design of high passenger-flows mass transit and flexible public transportation carrying low passenger-flows. PRT is one of such flexible system serving as a supplemental mode for TOD development, where a PRT system functions as a local area network, connecting the traditional transit systems and other means of transit modes within its network.

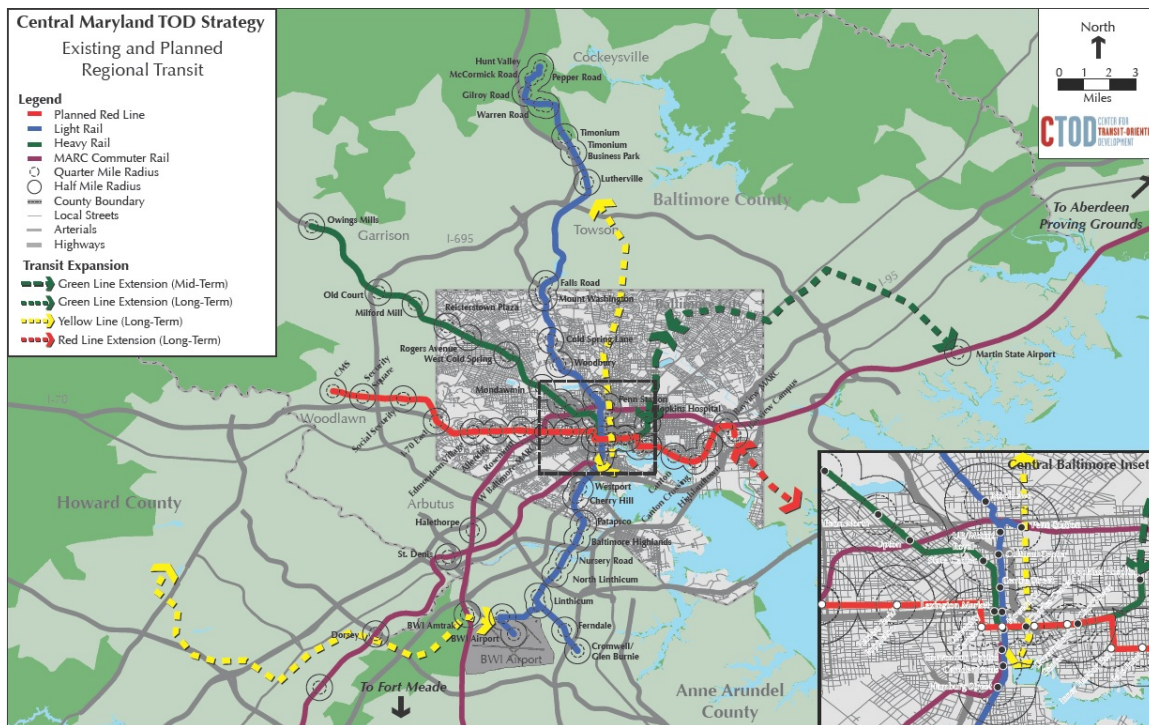


Figure 1.4 An example of TOD development in Maryland

(Source: (Central Maryland Transportation Alliance 2009))

PRT could be a sustainable solution to urban problems as well. Congestion in major cities results in not only severe travel time delay, but also excess energy use and emissions. PRT is one solution to reduce congestion on urban highways. Because PRT system is electrically powered, there is no emission, and thus overall energy and emission could be significantly reduced. Compared with automobiles, the benefit of energy saving for a PRT vehicle could be 75% less in general, and the benefit of CO₂ emission reduction could be more than 60% (Lowson 2003). Areas of land use are also reduced because of the small scale of the system, compared to the traditional road infrastructure.

While PRT has been recognized as an important component of alternate solutions to passenger cars in sustainable transportation systems in the future, it has not yet achieved wide-spread commercial deployment in US. Two major downsides that restrict the PRT in the practical stage are the cost and line capacity. Studies show that the construction cost of guideways is estimated between \$5–\$15 million per lane per mile;

of the stations about \$0.5–\$3 million per station, and of the vehicles about \$0.2–\$0.7 million per vehicle (in 1996 dollars) (Yoder, Weseman et al. 2000). Consider the PRT system at West Virginia University at Morgantown as a real example; it consists of 8.7 miles of guideways and 5 stations, involving a cost over \$126 million as of 1979 – about \$319 million in 2004 dollars (Sproule and Neumann 1991). A majority of the cost is for the guideway construction. In this context, reduction of the guideway length is critical to reduce the life-time construction cost.

The PRT may also have a limited line capacity as opposed to other public transportation systems. The line capacity is governed by allowable vehicle headways, which are further dictated by the safety requirement for a brick-wall stop. Since PRT is designed to provide the flexible door-to-door accessibility for a small group of passengers, the size of a PRT vehicle is small, accommodating up to 6 passengers. The current design of PRT speed is also relatively slow, due to not only the passengers' comfort level, but also the low design standard for guideways for purpose to reduce the construction costs. For instance, the design speed of the Mogantown PRT system is up to 30 miles per hour (Juster and Schonfeld 2013). To meet the safety standard, the design of headway could range from 8 to 15 seconds in the current practice (Juster and Schonfeld 2013)¹. The factors above lead to a limited line capacity of around 2,000 – 2,500 passengers per hour, which is less than for conventional public transit modes with higher fleet size and operating speed.

Both the cost and line capacity could be improved by an appropriate guideway network (GN) design, because a well-designed GN not only improves the connectivity and accessibility, but also provides more options in the route choice. Different from other public transportation modes generally running through a single line, a PRT system is usually designed as an interconnected system (or grid of guideways) with junctions. Since vehicles do not have to follow a pre-defined route, the system allows a PRT vehicle to flexibly select a route from a variety of routes in the network; and thus the overall throughput of the entire network could be improved (Carnegie and Hoffman

¹ Some studies argue that the headway could be reduced to as short as 3 seconds ideally. The standard, however, was never seen in operations of real-world PRT applications.

2007). This motivates the study of the PRT GN design investigated in this project, to reduce the GN length and maintain a desirable line capacity.

1.3 Objective

In this project we study the guideway network design for the personal rapid transit to support transit-oriented development. The guideway network design consists of two components, i.e., minimizing the guideway construction cost and users' travel time. In particular, we introduce a set of optional points, known as Steiner points, in the graph to reduce the guideway length. The model is formulated as a combined Steiner problem and assignment problem, and a Lagrangian relaxation based solution algorithm is developed to solve the problem. Numerical studies are performed on a realistic-sized network. We show that the proposed model and solution algorithm can solve the PRT guideway network effectively.

1.4 Organization

This report is organized as follows. Chapter 2 briefly discusses the overall methodology, particularly with focus on a theoretical mechanism on how to reduce the GN length effectively. In Chapter 3 we propose a multi-commodity flow formulation for the GN network design, and propose a solution algorithm based upon Lagrangian relaxation. In Chapter 4 we present a case study to demonstrate the effectiveness of the proposed methodology. Finally some concluding comments are provided in Chapter 5.

CHAPTER 2. METHDOLOGY

2.1 *Introduction*

PRT research has focused on network design (Ma and Schneider 1991; Won, Lee et al. 2006; Won and Karray 2008; Kornhauser 2009), capacity analysis (Lowson 2003; Johnson 2005; Schweizer and Mantecchini 2007; Lees-Miller, Hammersley et al. 2010; Mueller and Sgouridis 2011), and empty vehicle management to reduce the passengers' waiting time (Andreasson 1994; Andreasson 2003; Lees-Miller, Hammersley et al. 2010; Schweizer, Danesi et al. 2012). In this study we investigate the PRT GN design to support TOD deployment. That is, the model is multimodal and incorporates vehicle and transit networks and their interconnections, and integrates with other modes of public transit, commute bus, light rail, heavy rail, metro system, etc. The GN design involves two objectives that may conflict with each other. From the perspective of stakeholders, as the construction cost is proportional to the length of GN, one objective is to minimize the total GN length. From the user's standpoint, an efficient GN system needs to reduce the users' travel time, thus the second objective is to minimize the total passengers' in-vehicle travel time. These two objectives could conflict each other, leading to a bi-criteria network design problem, and the optimal solution is known as pareto-optimal. In this study we assign different weights to the two objectives to balance the two criteria, therefore we solve the optimal solution with the weighted objective function rather than solving the pareto-optimal solution.

Given a set of PRT stations (also called terminals) denoted by V , suppose the origin-destination (O-D) demand between the stations is given, the goal of GN design is to establish a graph $G = (N, A)$, where N denotes a set of nodes, A denotes a set of arcs

(also called links), connecting the PRT stations V subject to two objectives: (1) minimizing the total link length; and (2) minimizing the total system travel time experienced by travelers. The first objective is intended to minimize the construction and maintenance cost of GN, and the second objective is to minimize travel time and improve the level of service for users.

2.2 *Reducing the GN length*

We show a set of intermediate points that may not be strictly needed to design the connecting network (GN) could be useful to reduce the length. In graph theory, such intermediate points optional in network connectivity are known as the Steiner points, and the problem is known as the Steiner problem on the graph. If the connecting network is restricted to a tree, the GN design subject to the minimum arc length is known as the min-cost Steiner tree problem (STP), which is NP-hard (Dreyfus and Wagner 1971; Hakimi 1971; Winter 1987). Figure 2.1 shows an example to illustrate that Steiner points appear critical in reducing the length in the GN design. More specifically, Figure 2.1(a) demonstrates the minimal length required to connect the three stations is 6, where the distance between each pair of stations is 2. Simply introduce an intermediate (Steiner) point in the middle, which is optional in connecting the three stations. Figure 2.1(b) shows that the length could be reduced to 3, half of the former case.

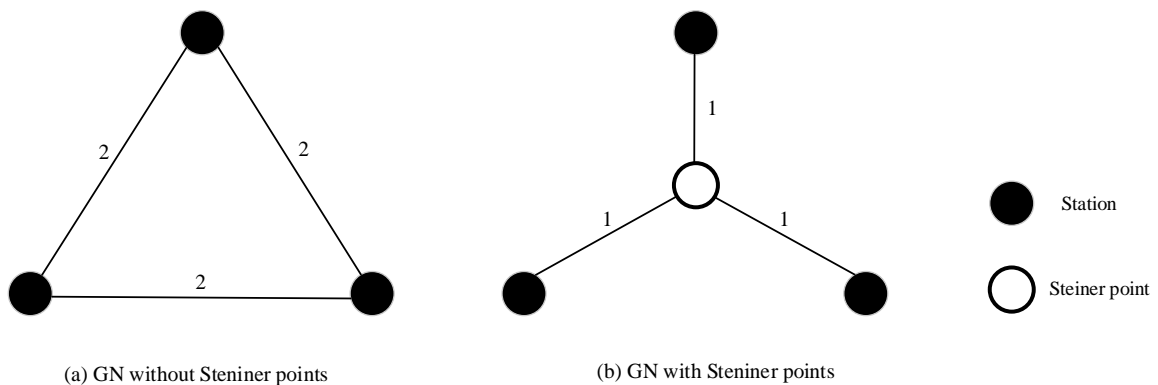


Figure 2.1 Steiner Points to Reduce the Network Length

The PRT GN design in this study leverages the concept of Steiner points, as they are essential in reducing the GN length effectively. Note that the PRT GN design in our problem is not restricted to a tree, hence the formulated model exhibits the Steiner point feature (i.e., optional connecting points) but the problem is not a STP. Further, since PRT guideways are subject to the line capacity representing the maximal vehicle per hour that can be afforded by the PRT service, arcs in the network must have capacity – another component differentiating our model from the STP.

2.3 Reducing the System Travel Time

For the PRT GN design, we introduce another objective from users' perspective to reduce the travel length by users. As a PRT vehicle travels at a constant speed in general, the PRT vehicle routing policy is the shortest distance (or travel time). Each link $(i, j) \in A$ is associated with a constant traversal time t_{ij} , and capacity u_{ij} representing the maximal hourly vehicular flow can pass the link. We then assign the O-D demand in the GN such that the system travel time is minimal while obeying the capacity constraint. This is a multi-commodity flow assignment problem.

The PRT GN design problem could be understood as a combined Steiner problem on the graph and min-cost multi-commodity flow assignment problem, as formulated in the next chapter.

CHAPTER 3. MODEL FORMULATION AND LAGRANGIAN-
RELAXATION-BASED SOLUTION ALGORITHM

3.1 Notation

A personal rapid transit (PRT) guideway network (GN) $G = (N, A)$ consists of a set of nodes N and a set of directed arcs A . A set of terminals (PRT stations) is denoted by $V \subseteq N$ that must be connected by A . A set of nodes $S = N - V$ is the set of Steiner points. We assume the geographical locations of S is given. Denote by c_{ij} the length, by t_{ij} the travel time, and by u_{ij} the capacity of arc $(i, j) \in A$, respectively. Flow assigned on arc (i, j) is denoted by x_{ij} . Denote the link construction decision by a binary variable $y_{ij} \in \{0,1\}$; $y_{ij} = 1$ if link (i, j) is constructed, $y_{ij} = 0$ otherwise.

Denote by P a set of origins and Q a set of destinations. A set of origin-destination (O-D) pairs is denoted by a vector $K := \{(p, q) \mid p \in P, q \in Q\}$. Denote by $k \in K$ a commodity associate with the O-D pair (p, q) . Let $p(k)$ represent the origin of commodity k and $q(k)$ represent the destination of commodity k . Let b^k be the corresponding O-D demand. Denote by x_{ij}^k the amount of commodity k assigned on arc (i, j) .

3.2 Multi-commodity Flow Formulation

The PRT GN problem is formulated as a multi-commodity flow problem as follows.

$$\min_{\mathbf{x}, \mathbf{y}} \quad \alpha \sum_{(i,j) \in A} c_{ij} \cdot y_{ij} + \beta \sum_{k \in K} \sum_{(i,j) \in A} x_{ij}^k \cdot t_{ij} \quad (1a)$$

$$\text{s.t.} \quad \sum_{(i,j) \in A} x_{ij}^k - \sum_{(j,i) \in A} x_{ji}^k = \begin{cases} b^k & i = p(k) \\ -b^k & i = q(k) \\ 0 & \text{otherwise} \end{cases} \quad \forall k \in K, \forall i \in N \quad (1b)$$

$$x_{ij}^k \leq y_{ij} \cdot b^k \quad \forall k \in K, \forall (i,j) \in A \quad (1c)$$

$$\sum_{k \in K} x_{ij}^k \leq u_{ij} \quad \forall (i,j) \in A \quad (1d)$$

$$M_1 \leq \sum_{(i,j) \in A} y_{ij} \leq M_2 \quad (1e)$$

$$x_{ij}^k \geq 0 \quad \forall k \in K, \forall (i,j) \in A \quad (1f)$$

$$y_{ij} \in \{0,1\} \quad \forall (i,j) \in A \quad (1g)$$

The first term of the objective function is to minimize total guideway length, or to minimize the construction (and maintenance) cost of guideways. The second term is to minimize users' total travel time. The overall objective function is balanced by assigning different weights, α and β , to the first and second term respectively. Eq. (1b) is flow mass balance constraint for each commodity k ; Eq. (1c) indicates that a commodity on arc (i,j) can be positive only if (i,j) is constructed (i.e., $y_{ij} = 1$), and it is zero if (i,j) is not constructed (i.e., $y_{ij} = 0$). Note that the amount of commodity k on an arc (i,j) is at most b^k . Constraints (1b)-(1c) indicates that a feasible solution must have a directed path of arcs (i.e., $y_{ij} = 1$) for each commodity $k \in K$. Thus we model the network connectivity via an embedded multi-commodity network flow problem. Eq.(1d) is the capacity constraint as the PRT guideway in general is subject to the maximal service rate. If the problem is uncapacitated, we assign u_{ij} a sufficiently large

number. To maintain connectivity between the terminal set V , Eq.(1e) indicates that the number of constructed arcs is bounded between $[M_1, M_2]$. To connect $|V|$ terminals it requires at least $|V| - 1$ arcs (i.e., terminal V is connected by a tree in a complete graph), so the lower bound of M_1 is $|V| - 1$. At most all arcs are constructed, so the upper bound of M_2 is $|A|$. Therefore initially one can set $M_1 := |V| - 1$, and $M_2 := |A|$. Eq.(1e) is redundant, however, it can be shown in the later discussion that this range could be tightened in the algorithmic procedure and thus provides a better bound (Beasley 1984). Finally Eq.(1f) specifies flow nonnegativity, and Eq.(1g) specifies a binary variable of y_{ij} .

To make the formulation stronger, we denote by f_{ij}^k the proportion of commodity k assigned on arc (i, j) , i.e., $f_{ij}^k = \frac{x_{ij}^k}{b^k}$, we can reformulate the problem as follows.

$$\min_{\mathbf{f}, \mathbf{y}} \quad \alpha \sum_{(i,j) \in A} c_{ij} \cdot y_{ij} + \beta \sum_{k \in K} \sum_{(i,j) \in A} f_{ij}^k \cdot t_{ij} \cdot b^k \quad (2a)$$

$$\text{s.t.} \quad \sum_{(i,j) \in A} f_{ij}^k - \sum_{(j,i) \in A} f_{ji}^k = \begin{cases} 1 & i = p(k) \\ -1 & i = q(k) \\ 0 & \text{otherwise} \end{cases} \quad \begin{matrix} \forall k \in K, \forall (i, j) \\ \in A \end{matrix} \quad (2b)$$

$$f_{ij}^k \leq y_{ij} \quad \begin{matrix} \forall k \in K, \forall (i, j) \\ \in A \end{matrix} \quad (2c)$$

$$\sum_{k \in K} f_{ij}^k \cdot b^k \leq u_{ij} \quad \forall (i, j) \in A \quad (2d)$$

$$0 \leq f_{ij}^k \leq 1 \quad \begin{matrix} \forall k \in K, \forall (i, j) \\ \in A \end{matrix} \quad (2e)$$

(1e, 1g)

Note that Eq.(2c) can be written into an aggregate constraint in Eq.(3). In the Lagrangian relaxation framework, dualizing Eq.(2c) involves $|K| \cdot |A|$ multipliers, and dualizing Eq.(3) leads to $|A|$ multipliers. As Eq.(2c) is stronger than the aggregated constraint (3), the former provides a better bound, which is a desirable feature in the algorithmic development. Therefore, we use the disaggregate constraint (2c) in the PRT model formulation.

$$\sum_{k \in K} f_{ij}^k \leq |K| \cdot y_{ij} \quad \forall (i,j) \in A \quad (3)$$

3.3 Lagrangian Relaxation

Eq.(2c) is the hard constraint coupling the flow assignment and network connectivity components. We dualize (2c) by associating a nonnegative Lagrangian multiplier $\boldsymbol{\lambda} = (\lambda_{ij}^k)_{k \in K, (i,j) \in A} \geq 0$, the Lagrangian dual problem (*LD*) is as follows.

$$LD: \max_{\boldsymbol{\lambda}} \min_{\mathbf{f}, \mathbf{y}} \sum_{(i,j) \in A} (\alpha \cdot c_{ij} - \sum_{k \in K} \lambda_{ij}^k) \cdot y_{ij} + \sum_{k \in K} \sum_{(i,j) \in A} (\beta \cdot t_{ij} \cdot b^k + \lambda_{ij}^k) \cdot f_{ij}^k \quad (4)$$

$$\text{s.t.} \quad (1e, 1g, 2b, 2d, 2e)$$

We intend to solve *LD* instead of solving the primal problem directly. Given a set of Lagrange multipliers, we call the sub-problem of *LD* the Lagrangian relaxation problem *LR* (formulated by Eqs. (5), (1e) – (1g), (2b) and (2d) – (2e)) with parameter $\boldsymbol{\lambda} \geq 0$. The goal is to solve the Lagrange multiplier $\boldsymbol{\lambda}$ that maximizes *LD*. This can be done by a sub-gradient algorithm if *LR* can be solved efficiently.

$$LR: \min_{\mathbf{f}, \mathbf{y}} \sum_{(i,j) \in A} (\alpha \cdot c_{ij} - \sum_{k \in K} \lambda_{ij}^k) \cdot y_{ij} + \sum_{k \in K} \sum_{(i,j) \in A} (\beta \cdot t_{ij} \cdot b^k + \lambda_{ij}^k) \cdot f_{ij}^k \quad (5)$$

$$\text{s.t.} \quad (1e, 1g, 2b, 2d, 2e)$$

Problem LR can be separated into two subproblems, $LR(1)$ and $LR(2)$, which are independent each other by inspection. $LR(1)$ is the network connectivity subproblem, and $LR(2)$ is the flow assignment subproblem. Below we show both problems could be solved trivially.

$$LR(1): \min_{\mathbf{y}} \sum_{(i,j) \in A} (\alpha \cdot c_{ij} - \sum_{k \in K} \lambda_{ij}^k) \cdot y_{ij} \quad (6a)$$

$$M_1 \leq \sum_{(i,j) \in A} y_{ij} \leq M_2 \quad (6b)$$

$$\text{s.t.} \quad y_{ij} \in \{0,1\} \quad \forall (i,j) \in A \quad (6c)$$

$$LR(2): \min_{\mathbf{f}} \sum_{k \in K} \sum_{(i,j) \in A} (\beta \cdot t_{ij} \cdot b^k + \lambda_{ij}^k) \cdot f_{ij}^k \quad (7a)$$

$$\text{s.t.} \quad \sum_{(i,j) \in A} f_{ij}^k - \sum_{(j,i) \in A} f_{ji}^k = \begin{cases} 1 & i = p(k) \\ -1 & i = q(k) \\ 0 & \text{otherwise} \end{cases} \quad \forall k \in K, \forall i \in N \quad (7b)$$

$$\sum_{k \in K} f_{ij}^k \cdot b^k \leq u_{ij} \quad \forall (i,j) \in A \quad (7c)$$

$$0 \leq f_{ij}^k \leq 1 \quad \forall k \in K, \forall (i,j) \in A \quad (7d)$$

$LR(1)$ could be easily solved by inspection. Denote a composite cost $c'_{ij} = \alpha \cdot c_{ij} - \sum_{k \in K} \lambda_{ij}^k$, we rank arcs in A according to the increasing order of c'_{ij} . Let r_{ij} represent the order of an arc (i,j) . The optimal solution of $LR(1)$ is

$$y_{ij} = \begin{cases} 1 & \text{if } r_{ij} \leq M_1 \\ 1 & \text{if } c'_{ij} \leq 0 \text{ and } M_1 < r_{ij} \leq M_2 \\ 0 & \text{otherwise} \end{cases} \quad (8)$$

Eq.(8) indicates that $y_{ij} = 1$ for the first M_1 arcs with the smallest c'_{ij} , and $y_{ij} = 1$ for the remaining arcs which admit $c'_{ij} \leq 0$.

$LR(2)$ can be easily solved too as it is simply a multi-commodity network flow problem which is a linear programming.

3.4 Solving the Lagrangian Dual Problem

The sub-gradient approach is a routine method to solve many Lagrangian dual problems. It generates a sequence $\lambda^0, \lambda^1, \dots, \lambda^n$ of Lagrange multiplier vectors heuristically following a descent direction. At each iteration, we solve the Lagrangian relaxation problem LR by solving $LR(1)$ and $LR(2)$ separately. Eq.(5) then provides an optimal solution of LR for a given λ , which is a lower bound to the primal problem denoted by Z_{LB} . Based on the solution of \mathbf{f} solve by $LR(2)$, we can also obtain a feasible solution of \mathbf{y} to the primal problem trivially, by setting $y_{ij} = 1 | (i,j) \in \mathbb{S}$, where \mathbb{S} denotes the arc set $\mathbb{S} = \{(i,j) | f_{ij}^k > 0, \forall (i,j) \in A, k \in K\}$. It implies that if an arc (i,j) carries positive flows for a commodity k , then (i,j) must be constructed in a feasible solution. The feasible solution \mathbf{y} and \mathbf{f} then provides an upper bound to the primal problem by Eq.(1a), denoted by Z_{UB} . Denote by $Z_{UB}^{min}, Z_{LB}^{max}$ the best upper and best lower bound that has been obtained by the algorithm, respectively. The duality gap is then specified by $GAP = Z_{UB}^{min} - Z_{LB}$. If GAP does not satisfy the stop criterion, the algorithm then generates a new Lagrange multiplier vector λ^{n+1} by a sub-gradient method. We apply the heuristic method proposed by Held and Karp (1971) to update λ^{n+1} :

$$\boldsymbol{\lambda}^{n+1} = \max\{\boldsymbol{\lambda}^n + \theta^n \frac{Z_{UB}^{min} - LD(\boldsymbol{\lambda}^n)}{\|s(\boldsymbol{\lambda}^n)\|^2} s(\boldsymbol{\lambda}^n), 0\} \quad (9)$$

where $LD(\boldsymbol{\lambda}^n)$ denotes an optimal solution to LR with a given $\boldsymbol{\lambda}^n$, θ^n is a step-size length (a scalar) with $0 \leq \theta^n \leq 2$, and $s(\boldsymbol{\lambda}^n)$ defines the sub-gradient direction specified by the relaxed equation (2c), which is:

$$s(\boldsymbol{\lambda}^n) = f_{ij}^k - y_{ij} \quad \forall (i, j) \in A, \forall k \in K \quad (10)$$

For the iterative technique used to determine successive values of $\boldsymbol{\lambda}$, the choice of step size θ^n strongly affects the convergence of LD . We choose the step size θ by $\theta = 5/\sqrt{n}$, where n denotes the iteration number.

We generate a sequence $\boldsymbol{\lambda}^0, \boldsymbol{\lambda}^1, \dots, \boldsymbol{\lambda}^n$ and compute Z_{LB}, Z_{UB} and GAP . Repeat the procedure until GAP converges to the stop criterion; we then terminate the algorithm with the best feasible solution. The sub-gradient algorithm solving the LD is presented as follows:

Algorithm: Solving the Lagrangian dual problem (LD) for the PRT GN design

- Step 1:* Set $n := 0$. Initialize a nonnegative dual vector $\boldsymbol{\lambda}^n$.
- Step 2:* Solve $LR(1)$ and $LR(2)$ with parameter $\boldsymbol{\lambda}^n$. Compute Z_{LB} by Eq.(5).
- Step 3:* Based on solution of $LR(2)$, construct feasible solution of \mathbf{y} ; compute Z_{UB} by Eq.(1a).
- Step 4:* Calculate GAP . If GAP satisfies the stop criterion, stop; otherwise go to Step 5.
- Step 5:* $n := n + 1$. Update $\boldsymbol{\lambda}^n$ by Eqs. (9) – (10), go to Step 2.

3.5 Generating Cuts to Tighten Lagrangian Lower Bound

In this section we discuss three cuts to tighten the Lagrangian lower bound. Denote by A_0 a subset of arcs that has been identified must not be constructed. Denote by A_1 a subset of arcs that has been identified must be constructed.

(1) Fix $y_{ij} = 0$

At each Lagrangian iteration when we solve $LR(1)$; let $c'' = \max_{(i,j) \in A} \{c'_{ij} | y_{ij} = 1\}$ and $c''' = \min\{c'_{ij} | y_{ij} = 0\}$. Recall that we scan arcs in the increasing order of c'_{ij} ; we then identify the first arc (v, w) such that $c'_{vw} - \max\{0, c''\} > Z_{UB}^{min} - Z_{LB}$. The set of arcs whose order is more than that of (v, w) , i.e., $\{(i, j) | r_{ij} \geq r_{vw}\}$, can be fixed by $y_{ij} = 0$ and added into A_0 .

This is because that suppose $y_{vw} = 1$, then the objective function of $LR(1)$ will increase. Consider the following two possibilities. (1) Suppose $c'' > 0$; it implies M_1 is binding; the objective function value will increase by $c'_{vw} - c''$. (2) Suppose $c'' \leq 0$; it implies M_1 is not binding; the objective function will increase by c'_{vw} . In both cases the new objective function will increase by $c'_{vw} - \max\{0, c''\}$. Suppose $y_{vw} = 1$ then the new Lagrangian lower bound equals $Z'_{LB} = Z_{LB} + c'_{vw} - \max\{0, c''\} > Z_{UB}^{min}$, which violates the Lagrangian duality theory and thus is not possible. The same result also applies to the set of arcs $\{(i, j) | r_{ij} \geq r_{vw}\}$, which can be added to A_0 and be eliminated from the problem. M_2 can be tightened by $M_2 := \min\{M_2, |A| - |A_0|\}$. \square

(2) Fix $y_{ij} = 1$

At each Lagrangian iteration when we solve $LR(2)$, we restrict $y_{ij} = 0$ one by one for each $(i, j) \in A - A_0 - A_1$ (the subset of arcs that has not been fixed) meanwhile $y_{ij} = 1$ in the solved solution \mathbf{y} . Let (v, w) be such an arc. We then calculate the optimal Lagrangian lower bound subject to $y_{vw} = 0$ as follows. For $LR(1)$, the penalty e_1 is subject to two possible conditions. If $\sum_{(i,j) \in A} y_{ij} > M_1$, there is $e_1 = -c'_{vw}$; otherwise $\sum_{(i,j) \in A} y_{ij} = M_1$, there is $e_1 = -c'_{vw} + c'''$. To compute the penalty for $LD(2)$, denoted by e_2 , we restrict $u_{vw} = 0$ and solve the restricted $LR(2)$ by LP; let Z_R

be the corresponding objective function value of the restricted $LR(2)$, and Z_2 be the objective function value of non-restricted $LR(2)$. The penalty of $LR(2)$ equals $Z_R - Z_2$. The addition of penalties imposed on both $LR(1)$ and $LR(2)$ is then compared with $Z_{UB}^{min} - Z_{LB}$. If $e_1 + e_2 > Z_{UB}^{min} - Z_{LB}$, we can fix $y_{vw} = 1$ and add (v, w) into A_1 . The logic is that suppose $y_{vw} = 0$; the new Lagrangian lower bound would be more than Z_{UB}^{min} and thus is impossible. M_1 can be tightened by $M_1 = \max\{M_1, |A_1|\}$.

(3) Penalties on number of arcs

At each Lagrangian iteration when we solve $LR(1)$, we solve the restricted problem by fixing $\sum_{(i,j)} y_{ij} = M_1$ and $\sum_{(i,j)} y_{ij} = M_2$, respectively. If the Lagrangian lower bound is more than the best upper bound Z_{UB}^{min} , it implies the restricted problem is infeasible; therefore we then can tighten M_1 or M_2 by 1.

(4) Problem reduction

Denoted by d_{ij} the shortest distance with respect to c_{ij} between nodes i and j . d_{ij} can be calculated by an all pairs shortest path algorithm, for instance, Floyd–Warshall algorithm. Any node $i \in N - V$ with $\min_{l \in V} d_{li} + \min_{j \in V} d_{ij} > Z_{UB}^{min}$ is not possible to be visited in an optimal solution, and thus can be eliminated from the problem.

CHAPTER 4. NUMERICAL EXAMPLE

In this chapter, we analyze a real-sized example to verify the PRT GN methodology and examine the algorithmic performance of the proposed method. The Lagrangian relaxation algorithm was coded in IBM ILOG CPLEX Optimization Studio 12.5.1 interfaced with MATLAB 2010b in Windows 7. Problem $LR(1)$ was solved by inspection; $LR(2)$ was a LP and solved by CPLEX12.1. We solve the PRT GN problem on a PC equipped with a 2.66-GHz Intel(R) Xeon(R) E5640 CPU with 24 GB of memory.

The network of the numerical example is multi-modal, and is illustrated in Figure 4.1. It contains 28 nodes and 43 arcs (two-way), with a total length 140 miles. It consists of 3 light rail stations, 13 PRT stations and 6 Steiner points. The size of the example is sufficiently larger than most PRT designs in the current practice. To build the network we suppose each mile of guideway costs \$10M. Characteristics of the network are tabulated in Table 1.

Table 4.1 Characteristics of the Network

Item	Characteristics
Guideway cost	\$10 million/mile
Operating speed	30 mile/hr
Capacity	1,000 passenger cars
Value of time	\$ 20 passenger/hr
α	0.5
β	0.5
K factor	0.2
Life cycle	10 years

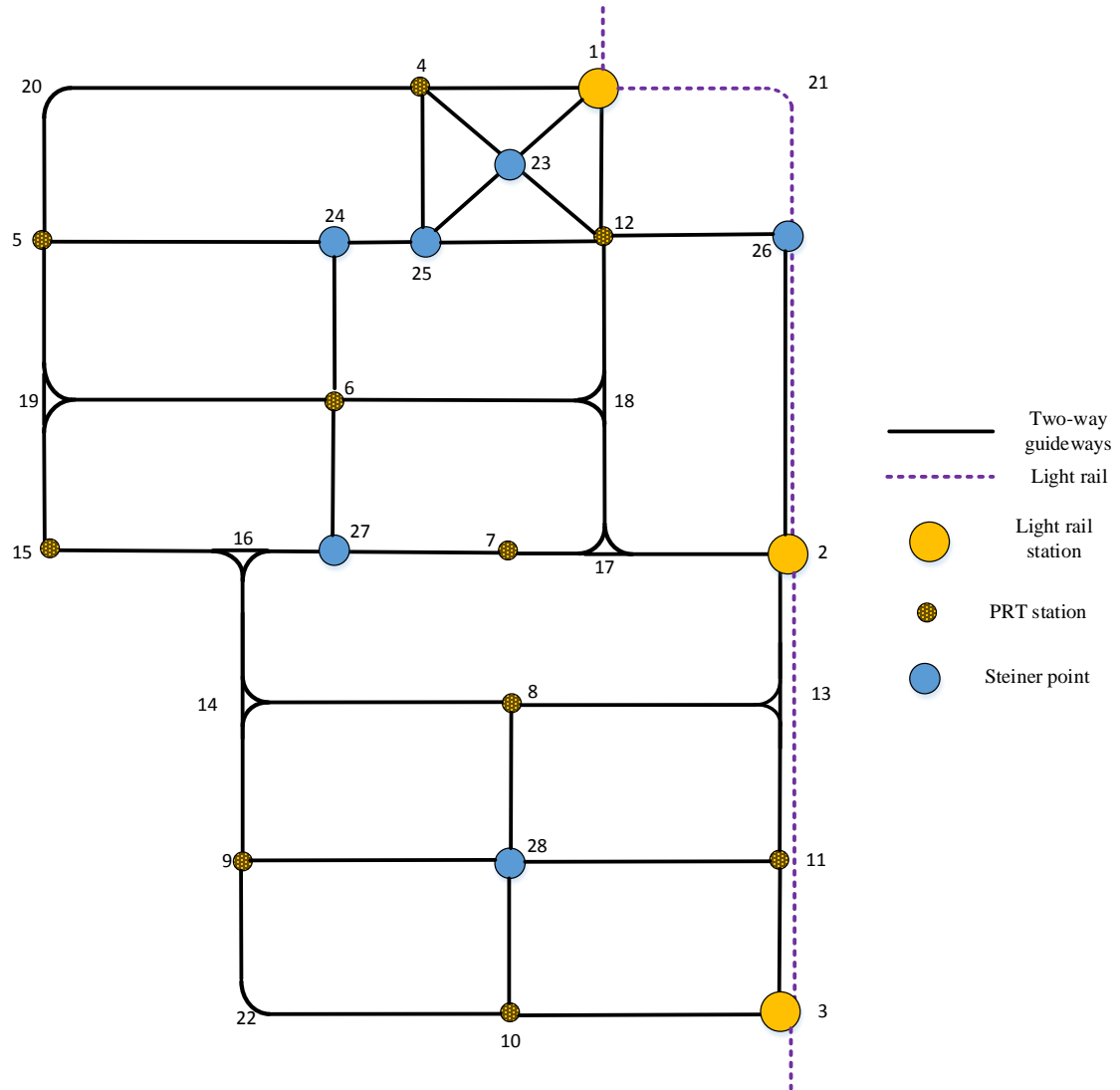


Figure 4.1 Network in the Numerical Example

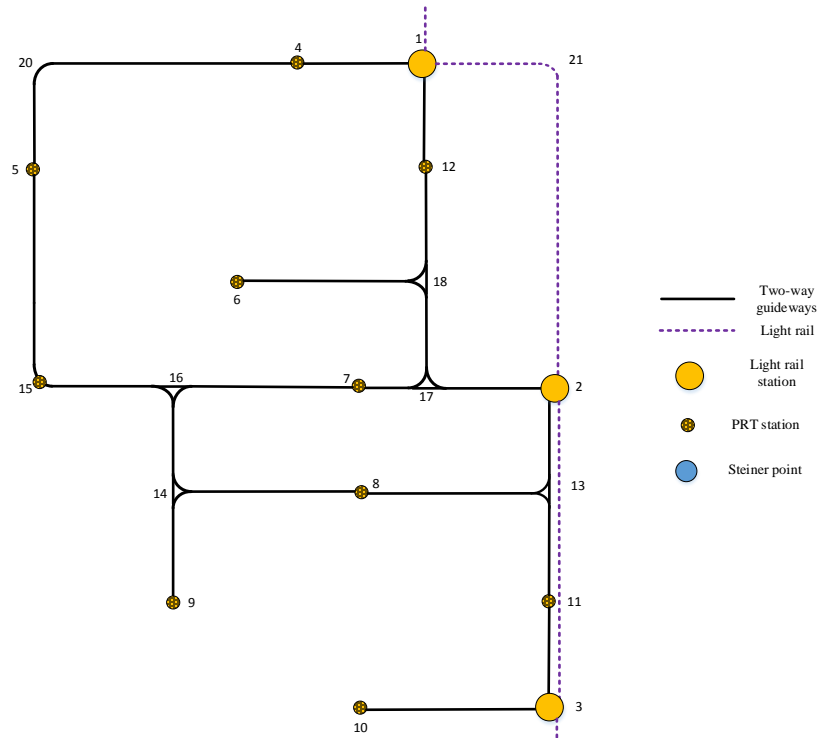
The hourly demand of the scenario is hypothetical. We assume that there are a significant amount passengers at the light rail stations due to the TOD. These passengers select the PRT mode to travel from or to their desired PRT stations by connecting through the commuter light rail. The total demand in the example is 3,790 passengers during the peak hour. The ratio of the peak hour demand and 24-hour demand (K factor) is assumed to be 0.2.

The objective function specifies $\alpha = 0.5$ and $\beta = 0.5$. It implies we place equal weight on the construction and users' costs. To compute users' costs we use value of time to convert the travel time to dollars, to make it comparable to the construction costs. In evaluation of the life-cycle travel time cost for the PRT, we calculate 10 years travel time.

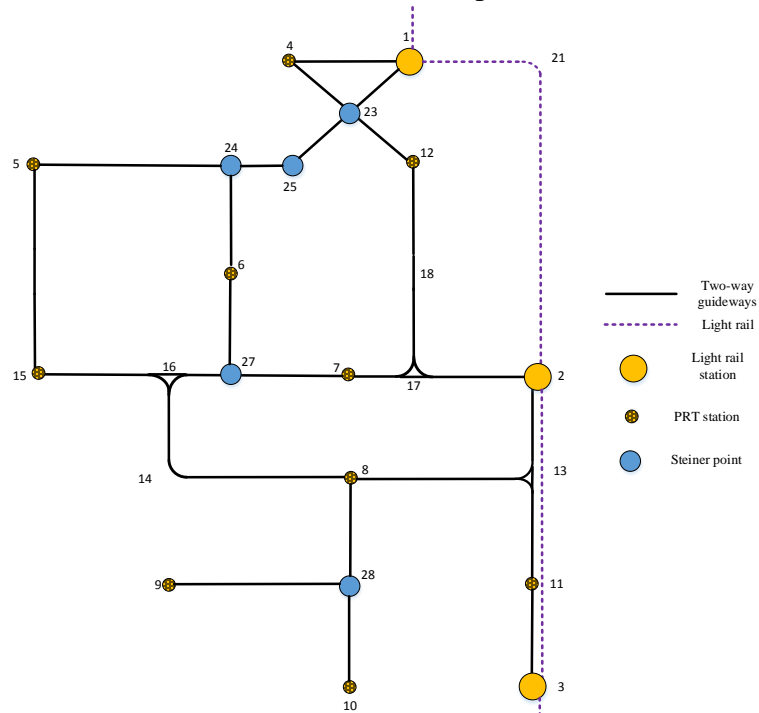
The proposed Lagrangian relaxation method can solve the two problems, with and without Steiner points, to a duality gap less than 1. The problems are deemed to be solved to optimality with such small duality gaps. The solutions are compared in Table 2, and the solved PRT guideways are plotted in Figure 4.2. Without Steiner points, the total guideway length is 42.7 mile. Considering that the construction cost is \$10 million per mile per lane, the construction cost (two lanes for both directions) is \$ 854.2 million. With Steiner points, the total guideway length is 34.4 mile, and the construction cost is \$687.4 million. The reduction is about 20%. Without Steiner points, the total user cost is \$187.6 million (life cycle is 10 years); with Steiner points, the total user cost is reduced to \$177.4 million. The improvement is about 5%. Hence, the introduction of Steiner points not only reduces the length (and investment) of the guideway, but also reduces the users' travel time significantly. Figure 4.2 demonstrates that the introduction of Steiner points can also lead to significant difference of the GN topology.

Table 4.2 Characteristics of the Solutions

	With Steiner points	No Steiner Points
Guideway length (mile)	34.4	41.8
Construction costs (million \$)	343.7	411.8
User costs (10 years, million \$)	177.4	187.6
UB	522.1	600.4
Lagrangian LB	521.1	599.4
Duality gap	0.9	1.0
Computational time (secs)	1689.1	33.5



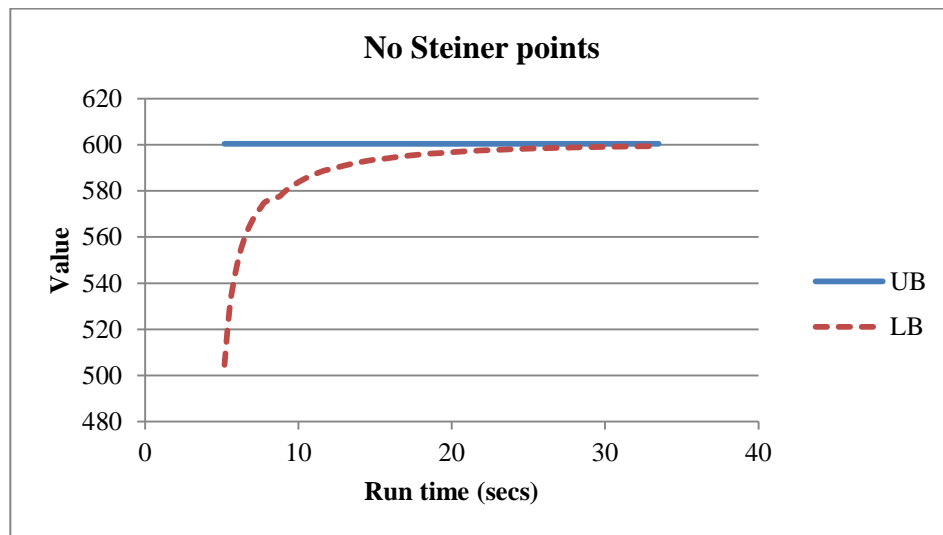
(a) with no Steiner points



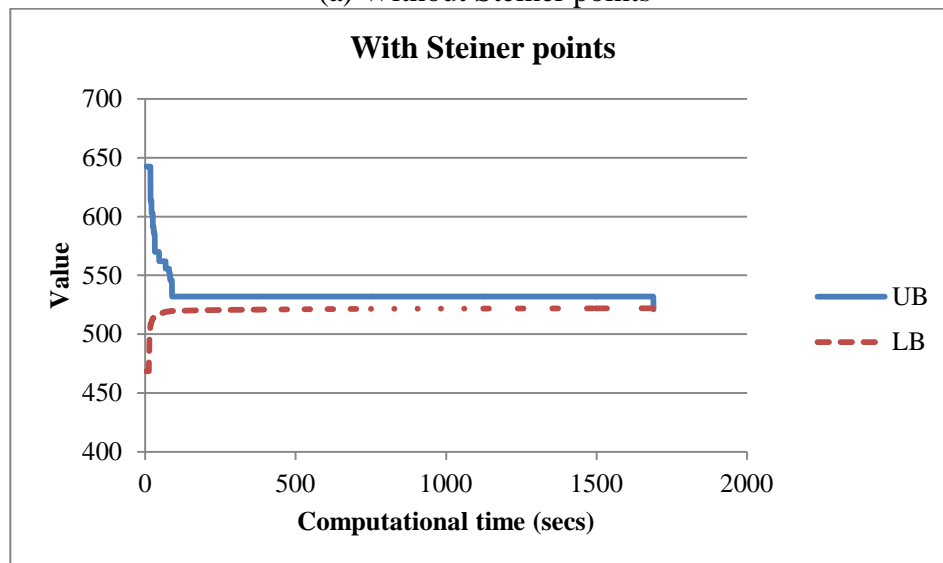
(b) with Steiner points

Figure 4.2 Guideway Networks Solved w/o Steiner Points

As for the computational performance, the scenario of with Steiner points takes up to 28 minutes to solve to optimal. It runs up to 2,970 iterations. The scenario of without Steiner points runs much faster. It takes up to 0.5 minutes to optimal, with 83 iterations. It indicates that the GN design with Steiner points is much harder to solve than the one without Steiner points. The proposed Lagrangian relaxation algorithm can solve both scenarios to optimal within reasonable amount of time. The convergence performance of the algorithm is plotted in Figure 4.3.



(a) Without Steiner points



(b) With Steiner points

Figure 4.3 Convergence Performance of the Lagrangian Relaxation Algorithm

CHAPTER 5. CONCLUDING COMMENTS

This chapter presents concluding comments on this research, highlights its significance, and suggests directions for future research.

5.1 *Summary and Conclusions*

In this study we present a model and solution algorithm for the GN design for the PRT supporting the transit oriented development. The GN is designed to balance two objectives, i.e., life-time construction cost and users' travel cost. We show the introduction of a set of optional points, known as Steiner points, can reduce the GN length, as well as the construction cost, substantially. The resultant GN design problem is then formulated as a combined Steiner problem and assignment problem. We propose a Lagrangian relaxation algorithm to decompose the Lagrangian dual problem into two subproblems; both are trivial to solve. We use a realistic-sized numerical example to demonstrate the computational performance, and validate the fact that a set of Steiner points can reduce both construction cost and users' travel cost significantly.

5.2 *Future Research*

The proposed model is for purpose of offline planning, or what-if scenario analysis, to assist PRT GN design to support TOD. On a large-scale network, the Lagrangian relaxation algorithm may entail a significant amount of time to solve to a desired duality gap. This is a theoretical feature of the algorithm because the complexity of the formulated model is NP-hard. In general, there is always a tradeoff between computational performance and solution quality. Nevertheless, we show the proposed

Lagrangian relaxation algorithm can solve a realistic-size PRT example to optimality in a reasonable amount of time.

The proposed model framework is tested on a hypothetical instance only; implementation of the proposed method to a real-world PRT GN design represents a future goal.

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